PHENOMENOLOGICAL MODELING OF PECULIARITIES OF INCLINED CRACK INITIATION

V. A. Dolgorukov and V. N. Shlyannikov

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Determining the direction of fracture initiation θ^* is an important element of solving problems on carrying capacity of structural components with crack-type defects, which are arbitrarily oriented in practice. The value of the angle θ^* has a profound effect on the nature of the fracture path and, as a consequence, on the life of structural components [1-3]. Therefore, estimation of θ^* as one of the aspects of criterion analysis according to boundary conditions of the problems has become a traditional line of study of crack mechanics. Such works are reviewed in [4, 5].

In the context of such investigations, however, there is a number of problems, which have yet to be represented in the literature. Precisely these problems are the object of the present work. Among them are conceptual grounding for the approaches to the problem solution and analysis of phenomenological aspect of the process, as compared to the experimental data.

The works dealing with estimation of the angle θ^* are based on the assumptions that the fracture direction determines an extreme value of a certain parametric characteristic. One of the specific aspects of deformation and fracture at the apex of a crack is localization of the process, determined by the singularity of stressed and deformed state. This is the reason why an adequate determination of θ^* cannot be obtained on the basis of only continuum mechanics without explicit regard to the material's structural properties.

With local approach by means of mechanics we can obtain (select) only the relations that describe the most general mechanisms predetermining the fracture direction. An extremal analysis of a certain function of θ and criterion investigation of the state by which we imply, in the context of engineering approaches, the comparison of actual and permissible values of the corresponding parameters within the scope of the crack problems are not connected directly. Therefore, it is not incidental that a comparison of the results based on different power, force, and deformation criteria and experimental data has shown that none of them describes completely the peculiarities of the materials in estimating the crack initiation direction. The problem of determining the fracture angle θ^* is not related to the problem of estimating the limiting state (loads). That is why the results given in [4, 5] can be treated as an investigation of some parametric features of the state of material at the crack's apex, as applied to estimating the expected fracture direction. This peculiarity of studying θ^* is emphasized and used within the limits of nonlinear analysis [6].

Another aspect of the problem consists in the assumption that θ^* determines the extreme value of only one parameter and this is insufficient for considering structural features of the process of damage accumulation and development. At the same time it is noted in [7] that a fracture only from normal stresses, as well as only from shearing stresses is almost impossible. This is because the limiting state takes place due to the material's ability to offer resistance to both shearing and normal stresses. Similar reasoning can also be extended to the grounds of any criterion obtained within classical mechanics. Therefore, phenomenological approach to the development of strength theories has great prospects for the limiting state of the material being considered.

It is shown in [8] that as regards the crack theory many phenomenological solutions are reduced to the Pisarenko—Lebedev strength theory [7]:

$$\varkappa\sigma_i + (1-\varkappa)\sigma_1 = \sigma_t \qquad (1)$$

($\kappa = \sigma_t/\sigma_c$, σ_t, σ_c are tensile and compressive strengths).

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The authors of [4, 9, 10] point out that this criterion may be applied to the problems of crack mechanics and in [9] they suggest to use within the framework of (1) the binomial expansion for σ_i and σ_1 in singular domain. However, a formal extension of minimax analysis [10] to the case of phenomenological criteria (criterion (1), in particular) does not take into account its physical essence and leads to incorrect situations.

Ratio (1) is developed especially for the criterion aspect of engineering strength theories upon complicated stressed state. This means that application of the left-hand part of (1) as a parametrical characteristic requires a special substantiation. On the other hand, as it was pointed out earlier, the traditional method of analyzing θ^* is based only on the functional relation between the corresponding expression and polar angle θ. Therefore the phenomenology of (1), determined by the application of different physical theories, results in the absence of a sufficient substantiation of the function

$$\frac{\partial}{\partial\theta}\left(\varkappa\sigma_i(\theta) + (1-\varkappa)\,\sigma_\theta(\theta)\right) = 0.$$

This can be avoided if we consider the peculiarities of conclusion (1), that consist in the assumption that the properties (state) of the material with regard to its micro-macro characteristics lie within the limits between the boundaries. The latter are obtained on the basis of the Mises theories and the first strength theory with interpolation functions with respect to κ. Hence it follows that for the problem under consideration the Pisarenko–Lebedev criterion should be written in the form

$$\varkappa\theta^*(\sigma_i) + (1-\varkappa)\,\theta^*(\sigma_\theta) = \theta^*. \tag{2}$$

It can be assumed that in such a statement we can consider, from a phenomenological point of view, both structural physical properties of the material by means of κ and multi-phase nature of the deformation and fracture process.

Confidence of ratios of the type (1) and (2) is determined by the corresponding estimates based on the selected theories and degree of representation of structural features on the basis of κ. As regards concepts, (2) must contain theories representing the whole possible range with respect to θ^* for a wide circle of materials. Therefore, under the conditions of limiting concentration of the stresses, in equations of type (1) and (2) other theories can be taken as base ones, whose analysis and development are being continued at present. Another problem, which has an independent significance proceeding from (2), is to estimate the possibility of applying the parmeter κ, as well as to determine the kind of functional relation on the basis of this characteristic.

The main requirement for investigating the influence of κ on the nature of variation of θ^* is the identity of external boundary conditions. This implies the equivalence of both loading conditions and geometrical factors of the samples being under experiments. The analysis is based on the experimental data obtained from a number of aluminum alloys upon cyclic biaxial loading [1]. The main characteristics are given in Table 1. The value of κ is obtained from reference data for σ_t and σ_c. Experimental results as regards the direction of crack initiation, according to its initial orientation α upon biaxial loading with biaxial ratio η are given in Fig. 1. Here the hatched lines show the range of variation of θ^* in the experiment for different loading conditions and the solid lines denote the curves $\theta^* = (\alpha, \eta)$, obtained from the simplest monomial variants of the strength theories contained in the base model (1). From Fig. 1 it follows that in order to solve the problem with regards to accumulation and development of damages in stages, it is necessary to find and use in (2) such variants of theories, which would determine a sufficiently wide (covering experimental data) range of upper and lower estimates of θ^*.

The influence of the values of κ is well to analyze, irrespective of the form of (2) concerning the theories used. Therefore, for the range with respect to θ^* we take the interval being observed in the experiment depending on the properties of the investigated materials. Further, for visual representation of the peculiarities of fracture initiation when the range of θ^* changes, a normalized axis θ^* must be taken. For each case of loading we take for 1 the upper value of the range of θ^* and for 2 we take the lower value, i.e., for $\eta = 0.5$, $\alpha = 25°$ 1 corresponds to $\theta^* = -49°$ (01419), 2) $\theta^* = -29°$ (AMg6). At the same time for $\eta = 0$, $\alpha = 65°$ 1 corresponds to $\theta^* = -30.2°$ (B95AT1), and 2) $\theta^* = -23°$ (AMg6). Then the results of the experiments [1] can be represented so as it is shown in Fig. 2. The line indicates the nature of interpolation functions with respect to κ in (2).

TABLE 1

Notation Fig. 1	Alloy	E	σ_b	$\sigma_{0.2}$	δ, %	n	$\varkappa$
		MPa					
1	AMg6	71	320	160	20,0	3,72	1,0
2	01420T	75	390	225	14,5	4,53	0,85
3	1163AT	72	439	285	19,5	4,94	0,85
4	D16ChAT	72	445	310	18,7	9,68	0,8
5	1201AT	71	420	320	13,0	3,71	0,7
6	1163ATBMO	72	478	369	12,2	5,14	0,7
7	B95AT1	72	563	506	11,0	10,05	0,5
8	01419	70	345	300	9,0	—	0,5

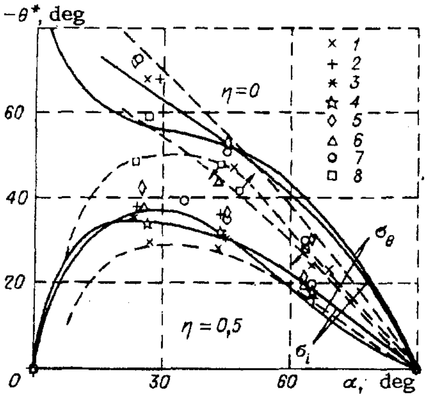

Fig. 1

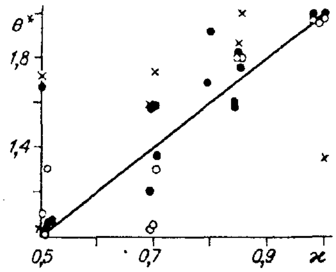

Fig. 2

Analyzing the data given in Fig. 2, the following circumstances should be taken into account. The results that belong to the greatest interval of variation of θ* are the most applicable for the present investigation. This occurs for η = 0.5, α = 25 and 45° (dark points in Fig. 2). The data obtained in uniaxial tension (light points) have a less model purity, which is explained by narrowing of the range over θ* (with respect to biaxial loading) and, as a consequence, by an increased influence of metrological errors and stochasticity of the process. Figure 2 demonstrates that the points' scattering increases with a narrowing of the range over θ*. The greatest scatter of the data is observed for η = 0.5, α = 65° (denoted by ×). Interpretation of the data of Fig. 1, with respect to characteristic of κ and the remarks, has made it possible to establish the fracture mechanism. It consists in that more plastic materials (AMg6) yield the least angle of deviation θ*, while more brittle materials yield the greatest θ*. In that case the latter (B95AT1, 01419) demonstrate a peculiar behavior with respect to the observed mechanism under the conditions of biaxial loading. Thus, for η = 0.5 the data for B95AT1 are in poor correlation with the model, while a diametrically opposite tendency is noted for 01419 upon uniaxial tension. Hence, on the whole, the represented results demonstrate a correlative interrelation between experimental data and model description of crack initiation direction.

From the investigation it was established that within local approach the deformation and fracture peculiarities upon inclined cracks initiation can be simulated by means of phenomenological relations of type (1). Here their specifics should be taken into account as is demonstrated by the discussion of the Pisarenko-Lebedev theory. It is shown that characteristic of the materials properties in the form of the ratio between tensile strength and compression strength when using it in relations of type (2), on the whole makes it possible to solve the problem on fracture direction and trajectory.

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DETERMINATION OF STRESS INTENSITY FACTORS AND CRACK-OPENING STRESSES FROM JUMPS IN CRACK-EDGE DISPLACEMENTS

V. N. Maksimenko

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In accordance with the superposition principle, stress-intensity factors (SIFs) – which control crack growth – can be calculated through the distribution of the nominal (crack-opening) stresses acting at the site of a crack in an undamaged structure. In actual structures, these stresses may differ appreciably from the values predicted theoretically. Various theoretical-experimental methods are used to determine the nominal stresses (cutting out layers, drilling holes, cutting notches [1-5]) and the SIF from the strain fields in the region ahead of a crack tip (extensometry, recording of the opening of the crack near its tip by means of sensors or computer analysis of visual images, laser-assisted speckle methods, photoelastic and holographic methods [4-10], etc.). However, in most cases these methods are flawed by several deficiencies: the time consumed in different stages of machining; limitations of computational procedures in regard to specific configurations; a certain arbitrariness and lack of rigor attending the use of the methods.

Using integral representations of solutions of problems concerning the elastic equilibrium of anisotropic plates weakened by a curvilinear slit (crack), here we proposed a method which makes it possible to use experimental jumps in crack-edge displacement found for several points to calculate two different stress-intensity factors and the distribution of the nominal stresses on the line of the crack in complex sectional structural elements made of metallic and composite materials. The efficiency and accuracy of the proposed approach is evaluated by mathematically modeling several problems of practical importance and comparing the results with experimental data.

1. We will take a loaded structure and isolate a plane element representing a plate made of an elastic, rectilinearly anisotropic (specifically, isotropic) material. The element occupies the finite region D in the plane xOy, and it contains a system of holes and cracks. The geometry of the structural element is shown in Fig. 1.

We will assume that the slit (crack) L passes completely through the element and that its edges are free of external forces. The structure is loaded in such a way that a plane stress state is realized in it in the absence of slit L.

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